Announcements:

1) HW #2 Supplement due tomorrow

2) Quiz Thursday over today's material.

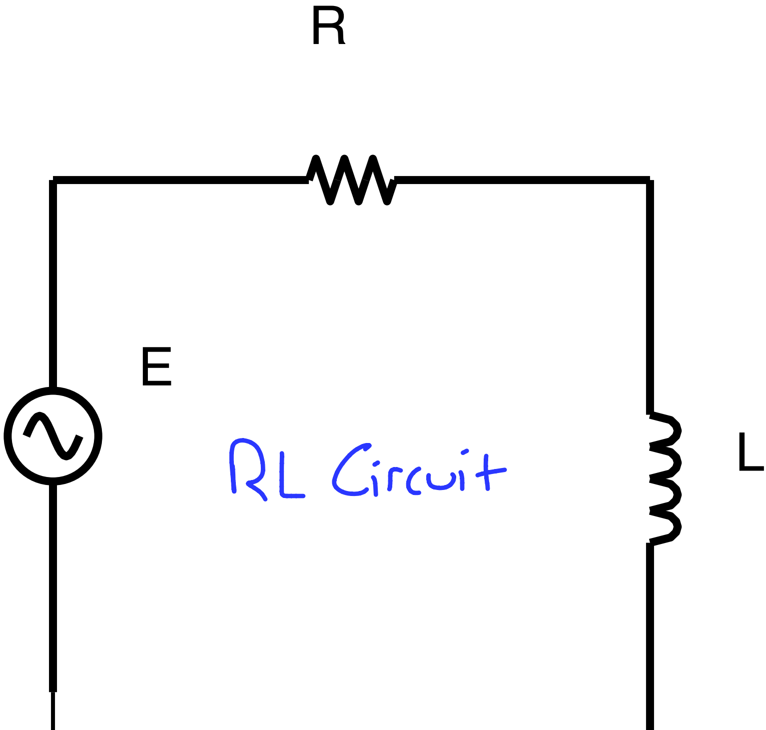
Circuit Problems

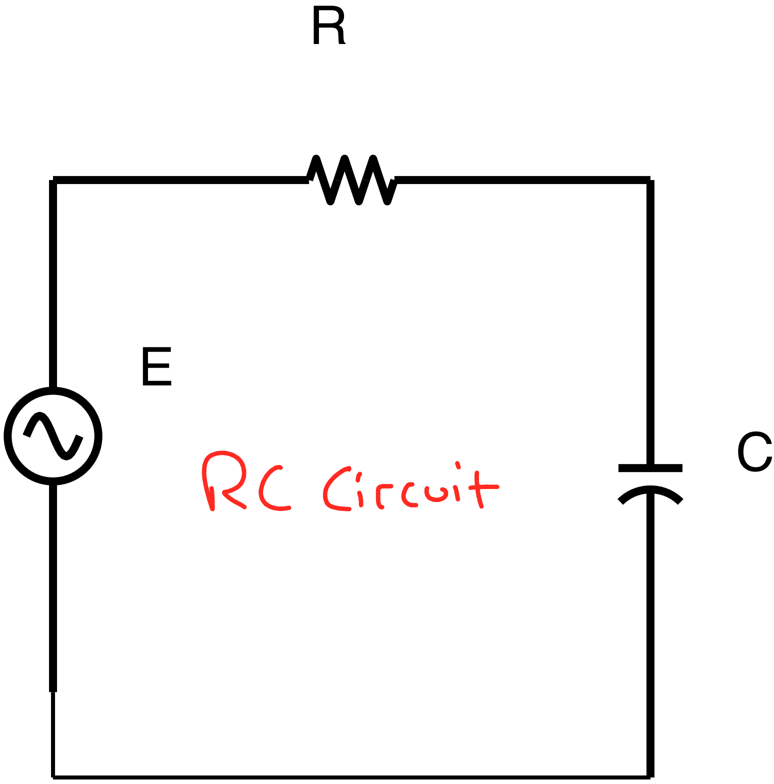
(section 3.5)

We will only consider simple circuits with a voltage source (battery or generator), a resistor, and either an inductor or capacitor (but not both!)

One inductor = RL Circuit

One Capacitor = RC Circuit


R
E
L
RL Circuit


R
E
C
RC Circuit

Here are a bunch of laws

1) Kirchoff's Current: the sum of all currents flowing into any junction point is zero
2) Kirchoff's Voltage: the sum of the instantaneous changes in potential (voltage drops) around any closed loop is zero

3) Ohm's Law : if E_R is the voltage drop through a resistor and I is the current,

$$E_R = RI$$

4) Faraday's/Lenz's Law :

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

where $\mathcal{E}$ = electromotive force,

Φ_B = magnetic flux

Using this, we can show

$$E_L = L \frac{dI}{dt}$$

where E_L is the voltage drop across an inductor

5) If q is the electrical charge on a capacitor,

$$E_C = \frac{1}{C} q$$

where E_C is the voltage drop across a capacitor

In an RL Circuit

Kirchoff's Voltage Law implies

$$E(t) = E_L + E_R$$

where $E(t)$ is the voltage in the circuit at time t

From Faraday's Law,

$E_L = L \dfrac{dI}{dt}$, substitute.

We get

$$E(t) = L\frac{dI}{dt} + E_R.$$

Using Ohm's Law,

$$\boxed{E(t) = L\frac{dI}{dt} + RI(t)}$$

Can solve using integrating factors.

Example 1: If $R = 1\Omega$, $L = .01$ H (Henry) and $E(t) = \sin(100t)$, solve for I in our equation

$$E(t) = L\frac{dI}{dt} + RI(t).$$

We have

$$\sin(100t) = .01 \frac{dI}{dt} + I(t).$$

Rewrite in standard form:

$$100 \sin(100t) = \frac{dI}{dt} + 100 I(t).$$

Multiply both sides by the integrating factor $\mu(t)$ where

$$\frac{d}{dt}\left(\mu(t)I(t)\right)$$

$$= \mu(t)\frac{dI}{dt} + 100\mu(t)I(t)$$

Solve for $\mu(t)$: By the product rule,

$$\mu'(t) = 100\mu(t),$$

so $\boxed{\mu(t) = e^{100t}.}$

multiplying by $e^{100t} = u(t)$,

$$100e^{100t}\sin(100t)$$

$$= e^{100t}\frac{dI}{dt} + 100e^{100t}I(t)$$

$$= \frac{d}{dt}\left(e^{100t}I(t)\right).$$

Integrate both sides with respect to t.

Integrating, we get

$$\int 100e^{100t} \sin(100t)\,dt$$

$$= \int \frac{d}{dt}\left(e^{100t} I(t)\right) dt$$

$$= e^{100t} I(t) + C.$$

How to solve

$$\int 100e^{100t} \sin(100t)\,dt\,?$$

$$\int 100 e^{100t} \sin(100t)\, dt$$

$u = 100t$

$du = 100dt$ gives

$$\int e^{u} \sin(u)\, du.$$

Integrate by parts:

$w = \sin(u)$ $\quad v = e^{u}$

$dw = \cos(u)du$ $\quad dv = e^{u} du$

We get

$$\int e^u \sin(u)\,du$$

$$= e^u \sin(u) - \int e^u \cos(u)\,du$$

Persevere! Stick to your choices of w, v.

$$w = \cos(u) \qquad v = e^u$$

$$dw = -\sin(u)\,du \qquad dv = e^u\,du$$

Don't choose $w = e^u$!

$$\int e^u \sin(u)\,du$$

$$= e^u \sin(u) - \left(e^u \cos(u) + \int e^u \sin(u)\,du\right)$$

$$= e^u\left(\sin(u) - \cos(u)\right) - \int e^u \sin(u)\,du$$

Add $\int e^u \sin(u)\,du$ to both sides!

We get

$$2\int e^{u}\sin(u)\,du$$

$$= e^{u}\left(\sin(u)-\cos(u)\right),$$

so
$$\boxed{\int e^{u}\sin(u)\,du = \frac{e^{u}\left(\sin(u)-\cos(u)\right)}{2}}$$

Substituting $u = 100t$,

$$\frac{e^{100t}\left(\sin(100t) - \cos(100t)\right)}{2}$$

$$= \int 100 e^{100t} \sin(100t)\, dt$$

$$= e^{100t} I(t) + C$$ from before

Using initial condition $I(0) = 0$

Using initial condition $I(0)=0$,

$-\frac{1}{2}=C$, and so

$$\boxed{I(t)=\frac{\sin(100t)-\cos(100t)}{2}+\frac{1}{2e^{100t}}}$$

Capacitors are Easier!

The our equation

$$E(t) = E_R + E_C$$

$$= RI(t) + E_C$$

and substituting

$$E_C = \frac{q}{C} ,$$

$$E(t) = R\,I(t) + \frac{q}{C}$$

There are no derivatives, so this is not a calculus problem!